Scientific Calculators allowed.

For #1 – 4, use $f(x) = -2x^2 + 8x + 10$.

1) Write $f(x)$ in vertex form by completing the square.

Start with: $f(x) = -2x^2 + 8x + 10$

Move the constant to the right: $= -2x^2 + 8x \quad + 10$

Factor -2 out of the first two terms: $= -2(x^2 - 4x + \_\_\_\_) \quad + 10$

Add and subtract: the square of half of the coefficient of x: $\left(\frac{-4}{2}\right)^2 = 4$: $= -2(x^2 - 4x + 4) \quad + 10 + 8$

Notice that we are adding $(-2 \cdot 4 = -8)$ on the left,
So we need to subtract -8 (i.e., add 8) on the right

Express the parenthetical term as a square and collect the constants on the right: $= \mathbf{-2(x-2)^2 + 18}$

2) Find the coordinates of all intercepts.

To find the x-intercepts, set $y = 0$ and solve for x.

$0 = -2x^2 + 8x + 10$

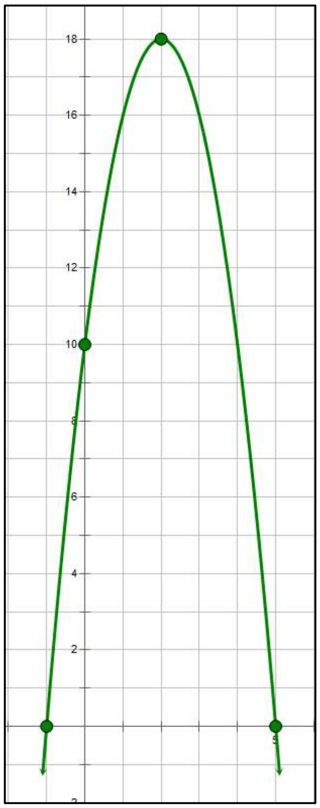
$= -2(x^2 - 4x - 5)$

$= -2(x+1)(x-5)$, so, $x = -1, 5$, and **the x-intercepts are: $(-1, 0), (5, 0)$**

To find the y-intercepts, set $x = 0$ and solve for y.

$y = -2(0)^2 + 8(0) + 10 = 10$, so **the y-intercept is: $(0, 10)$**

Note, when a quadratic equation is expressed in standard form: $y = ax^2 + bx + c$, the y-intercept always occurs at the constant term, so the y-intercept is always: $(0, c)$.



3) Graph $f(x)$ on the provided coordinate system. Include the vertex and all intercepts.

To graph $f(x)$, use the results of problems 1 and 2.

From problem 1, the vertex is $(2, 18)$. Also, notice that the sign in front of the x-term is negative, so the quadratic opens down.

From problem 2, we have three more points on the curve: the x- and y-intercepts. Plot these points and run a smooth curve through the points.

For some problems, additional points may be needed to plot the curve. In that case, it is a good practice to select x-values one or two units to the left and right of the vertex, then calculate the associated y-values and plot the points.

For #4 – 5, use $g(x) = \frac{1}{2}(x-4)^2 - 6$.

4) Find the requested information for $g(x)$ in the table below. If needed, round to one decimal place.

Vertex: $(4, -6)$	Opens up or down? **opens up**
Axis of symmetry: $x = 4$	x −intercepts (if any): $(0.5, 0), (7.5, 0)$
y −int: $(0, 2)$	max or min? **minimum** Value: -6
Domain (interval notation): $(-\infty, \infty)$	Range (interval notation): $[-6, \infty)$

$g(x)$ is given in vertex form: $g(x) = a(x-h)^2 + k$, where (h, k) is the vertex, so the vertex is: $(4, -6)$.

The curve will **open up** because the coefficient of the x-term is positive.

The axis of symmetry always runs vertically through the vertex. Its equation is always: $x = h$, so for this problem, the axis of symmetry is: $x = 4$.

To find the x-intercepts, set $y = 0$ and solve for x.

$$0 = \frac{1}{2}(x-4)^2 - 6$$

$$6 = \frac{1}{2}(x-4)^2$$

$$12 = (x-4)^2$$

$$\pm\sqrt{12} = x - 4$$

$$4 \pm \sqrt{12} = x$$

$x \sim 0.5, 7.5$, so **the x-intercepts are approximately** $(0.5, 0), (7.5, 0)$

To find the y-intercepts, set $x = 0$ and solve for y.

$y = \frac{1}{2}(0-4)^2 - 6 = 2$, so **the y-intercept is:** $(0, 2)$

To determine the max, min, domain, and range, it helps to graph the quadratic, which is what problem 5 asks us to do. So let's do problem 5 before determining these. See the graph below.

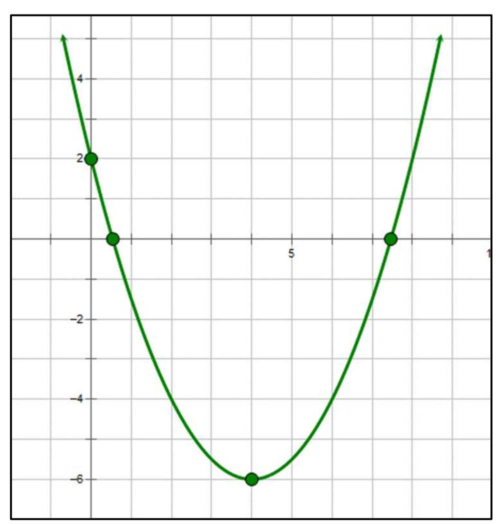
A function that opens up has a **minimum**, the value of which is $k = -6$, the y-value of the vertex. Maxima and minima always occur at the vertex.

The **domain** of a quadratic function is always all real numbers: $(-\infty, \infty)$ **or** $\mathbb{R}$.

The **range** can be taken from the graph. It is always: $[k, \infty)$ if the curve opens up, or $(-\infty, k]$ if the curve opens down. For this problem, the range is: $[-6, \infty)$

5) Graph $g(x)$ on the provided coordinate system. Include the vertex and any intercepts.

See the graph plotted to the right.
The vertex and intercepts are shown.



6) Joseph has started a company that makes mountain bikes. The profit (in \$) from selling x bikes can be found by using $P(x) = -200x^2 + 92000x - 8{,}400{,}000$. What is the max profit that his company can earn?

Recall that the maximum value of a quadratic function is k, the y-value of the vertex. Let's put this equation in vertex form to find the vlaue of k.

$$P(x) = -200x^2 + 92000x \qquad - 8{,}400{,}000$$
$$= -200(x^2 - 460x + \_\_\_\_) - 8{,}400{,}000$$
$$= -200(x^2 - 460x + 230^2) - 8{,}400{,}000 + 200(230)^2$$ profit is in purple

$$-8{,}400{,}000 + 230^2 = \$\mathbf{2,180,000}$$

7) Consider $h(x) = -3x(x+2)^2(x-4)$. Which the statements below are true? **Select all that apply.**

A) $h(x)$ has zeros at $x = -2, 0,$ and 4
B) The zero at $x = 4$ has a multiplicity of 2.
C) As $x \to \infty, h(x) \to -\infty$
D) As $x \to -\infty, h(x) \to -\infty$
E) The end behavior for $h(x)$ is different on the left and right sides.
F) $h(x)$ *crosses* the x-axis exactly twice.

What can we tell from the equation?

1. The zeros exist where $x = 0, x + 2 = 0, x - 4 = 0$, so, when $x = 0, -2, 4$.
2. Multiplicities are 1 for $x = 0,\ 4$, and 2 for $x = -2$. Multipicities are equal to the exponents of the terms.
3. Odd multipicities result in a curve that crosses through the x-axis, and even multiplicities result in a curve that bounces off the x-axis. So, this curve crosses @ $x = 0,\ 4$, and bounces @ $x = -2$.
4. The degree of the function is 4, so its graph is likely to have 3 humps, and a W-shape.
5. The negative in front of the equation means the shape will be an inverted W, like an M. This implies that $h(x)$ tends to $-\infty$ on both the right and the left sides.

Based on these observations, we have:

A. True (observation 1)
B. False (observation 2)
C. True (observation 5)
D. True (observation 5)
E. False (observation 5)
F. True (observation 3)

So, Answers A, C, D, F are true.

For #8 – 9: Find the zeros and give the multiplicity for each zero, as needed.

8) $y = x^3 - x^2 - 9x + 9$

$$y = x^2(x-1) - 9(x-1)$$
$$= (x^2 - 9)(x - 1)$$
$$= (x-3)(x+3)(x-1)$$

Zeros are: $x = -3, 1, 3$
Each zero has multiplicity 1

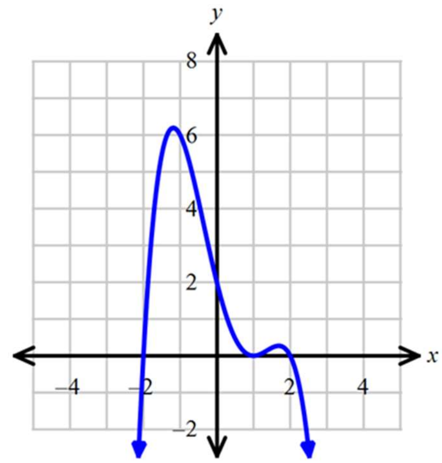
9) $g(x) = x^3 + 8x^2 + 16x$

$$g(x) = x(x^2 + 8x + 16)$$
$$= x(x+4)^2$$

Zeros are: $x = 0$ with multiplicity 1
$x = -4$ with multiplicity 2

10) Alexis, Pattra, and Adrian were working on a problem together in math where they had to match a graph with its equation.

- Alexis believes the graph is for $f(x) = -\frac{1}{2}(x-1)^2(x+2)$.
- Pattra believes the graph is for $h(x) = -\frac{1}{2}(x-1)^2(x-2)(x+2)$.
- Adrian believes the graph is for $g(x) = -\frac{1}{2}(x-1)^2(x+2)^2$.
- Who is correct, and how do you know?



This equation has 3 humps, so it is likely to have degree 4. This means Alexis is incorrect.

The curve crosses the x-axis at $x = -2, 2$, so those zeros have odd multiplicities, probably 1. This means Adrian is incorrect.

The curve bounces off of the x-axis at $x = 1$, so $x = 1$ has an even multiplicity, probably 2.

Pattra is correct because the equation: $h(x) = -\frac{1}{2}(x-1)^2(x-2)(x+2)$ has the correct multiplicities for each zero. The others do not. Also, note that the curve opens down, approaching $-\infty$ on both sides, which is characteristic of an even degree function with a negative lead sign.

11) Which of the following statements are true for $f(x) = 3x^4 + 2x^5 + 17x^6 - 3$? **Select all that apply.**

A) $f(x)$ has 6 total zeros (real and imaginary combined)
B) $f(x)$ has 4 total zeros (real and imaginary combined)
C) as $f(x) \to \infty, f(x) \to \infty$
D) as $f(x) \to \infty, f(x) \to -\infty$
E) as $f(x) \to -\infty, f(x) \to \infty$
F) as $f(x) \to -\infty, f(x) \to -\infty$

Rewrite $f(x)$ from highest degree to lowest degree: $f(x) = 17x^6 + 2x^5 + 3x^4 - 3$

What can we tell from the rewritten equation?

1. $f(x)$ is of degree 6, so it has 6 total zeros, including any duplicates.
2. The degree of the function is 6, so it could have as many as 5 humps, and a W-shape.
3. The positive in front of the leading x-term means the shape will be a regular W, not an inverted W. This implies that $f(x)$ tends to ∞ on both the right and the left sides.

Based on these observations, we have:

A. True (observation 1)
B. False (observation 1)
C. True (observation 3)
D. False (observation 3)
E. True (observation 3)
F. False (observation 3)

So, Answers A, C, E are true.

12) Find the quotient: $(2x^4 - 11x^3 + 8x^2 + 15x - 8) \div (x^2 + 1)$

$$
\begin{array}{r|l}
 & \mathbf{2x^2 - 11x + \;\; 6 + \frac{26x-14}{x^2-1}} \\
\hline
x^2 + 1 & 2x^4 - 11x^3 + 8x^2 + 15x - \;\; 8 \\
 & -\;(2x^4 \qquad\quad + 2x^2 \qquad\qquad\quad) \\
\hline
 & \qquad -11x^3 + 6x^2 + 15x - \;\; 8 \\
 & \qquad -\;(-11x^3 \qquad\;\; - 11x \qquad) \\
\hline
 & \qquad\qquad\qquad 6x^2 + 26x - \;\; 8 \\
 & \qquad\qquad\quad -\;(6x^2 \qquad\quad + \;\; 6) \\
\hline
 & \qquad\qquad\qquad\qquad\quad 26x - 14
\end{array}
$$

13) List all *possible* rational roots for $y = 3x^4 - 7x^2 + 8x + 10$.

All possible rational roots are ratios of factors of the constant, 10, divided by factors of the lead coefficient, 3.

Since the numerator and denominator can both be either positive or negative, placing a $\pm$ sign in front of the positive ratios will suffice.

$$\text{Possible roots} = \pm\frac{\text{factors of 10 } (1, 2, 5, 10)}{\text{factors of 3 } (1, 3)} = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$$

$$\text{Simplified rations are: } \pm\mathbf{1}, \pm\mathbf{2}, \pm\mathbf{5}, \pm\mathbf{10}, \pm\mathbf{\frac{1}{3}}, \pm\mathbf{\frac{2}{3}}, \pm\mathbf{\frac{5}{3}}, \pm\mathbf{\frac{10}{3}}$$

14) Given that $g(x) = 3x^3 - 5x^2 - 6x + 8$ and 2 is a zero for $h(x)$, then solve $h(x) = 0$ for all solutions.

Let's approach this with synthetic division.

$$
\begin{array}{r|rrrr}
2 & 3 & -5 & -6 & 8 \\
 & & 6 & 2 & 8 \\
\hline
 & 3 & 1 & -4 & 0 \\
 & x^2 & x & 1 & \text{remainder}
\end{array}
$$

At this point, we have:

$$3x^3 - 5x^2 - 6x + 8 = (x - 2)(3x^2 + x - 4)$$

Continuing:

$$3x^3 - 5x^2 - 6x + 8 = (x - 2)(3x + 4)(x - 1)$$

So, the zeros are: $\boldsymbol{x = -\frac{4}{3}, 1, 2}$

Tip: If a factored term of a polynomial is $(ax + b)$, the corresponding zero is: $-\frac{b}{a}$.

15) **Multiple Choice.** Use Descartes' Rule of Signs to determine the possible number of positive and negative real zeros for $f(x) = x^5 - 1.5x^4 - 13.76x^3 + 3x^2 + 34.42x - 15.397$.

A) 2 or 0 positive zeros; 3 or 1 negative zeros
B) 3 or 1 positive zeros; 3 or 1 negative zeros
C) 3 or 1 positive zeros; 2 or 0 negative zeros
D) 2 or 0 positive zeros; 2 or 0 negative zeros

The equation with highlighted signs is: $+x^5 - 1.5x^4 - 13.76x^3 + 3x^2 + 34.42x - 15.39$

For positive real zeros, look at the sign pattern of the coefficients, and see how many changes there are:

$+ \; - \; - \; + \; + \; -$ There are 3 sign changes, so there are 3 or 1 positive zeros.

For negative real zeros, change the signs associated with odd powers of x:

$- \; - \; + \; + \; - \; -$ There are 2 sign changes, so there are 2 or 0 negative zeros. **Answer C**

16) Find all rational zeros for $f(x) = 3x^3 - 19x^2 + 30x - 8$.

Time-saving tips:

- If the sum of the coefficients is 0, then 1 is a zero of the polynomial.
- If the sum of the coefficients of the odd-exponent terms is equal to the sum of the coefficients of the even-exponent terms, then -1 is a zero of the polynomial.

We will use synthetic division to find the first rational zero of this polynomial. Let's be clever about the zeros we try. Usually, this process will only take seconds, and will save us a lot of time.

- The sum of the coefficients is: $3 - 19 + 30 - 8 = 6 \neq 0$, so 1 is not a zero of this polynomial.
- The alternating sums of even and odd exponent terms are: $3 + 30 = 33$ and $-19 - 8 = -27$. These are not equal, so -1 is not a zero of this polynomial.
- There are 3 sign changes in the above equation, so there are 3 or 1 positive real roots. Let's try a positive root. The constant term is 8, so 2 and 4 seem like the most likely roots.

$$\begin{array}{r|rrrr} 2 & 3 & -19 & 30 & -8 \\ & & 6 & -26 & -8 \\ \hline & 3 & -13 & 4 & 0 \end{array}$$

At this point, we have:

$$3x^3 - 19x^2 - 30x - 8 = (x-2)(3x^2 - 13x + 4)$$

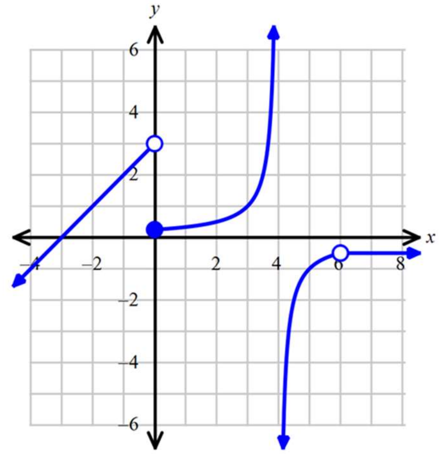
Continuing:

$$3x^3 - 19x^2 - 30x - 8 = (x-2)(3x-1)(x-4)$$

So, the zeros are: $\boldsymbol{x = \frac{1}{3}, 2, 4}$

Tip: If a factored term of a polynomial is $(ax + b)$, the corresponding zero is: $-\frac{b}{a}$.

17) Given the graph of $f(x)$ as shown, identify the x-values for each discontinuity. Classify each discontinuity as removable or non-removable, and describe it as either a hole, vertical asymptote (infinite discontinuity), or a jump discontinuity.



Discontinuities exist at:

- $x = 0$, which is a non-removable jump discontinuity.
- $x = 4$, which is a non-removable vertical asymptote (an infinite discontinuity).
- $x = 6$, which is a removable discontinuity (a hole).

Note: A removable discontinuity is a discontinuity where the limit exists, but the limit is not equal to the value of the function at that point. Therefore, only holes produce removable continuities. All other discontinuities are non-removable.

18) Find the equations of all asymptotes for $y = \frac{3x^2-8x+1}{x+4}$

The first step is to factor and reduce the rational function, if possible.

The numerator is factorable if its discriminant $(\Delta = b^2 - 4ac)$ is a perfect square. The discriminant is the expression under the radical in the quadratic equation.

For: $3x^2 - 8x + 12$, $\Delta = (-8)^2 - 4(3)(12) = -80$. Since -80 is not a perfect square, the numerator cannot be factored. This means the denominator will not change when the function is factored and simplified.

There is a vertical asymptote wherever the factored and simplified denominator is 0. So, **there is a vertical asymptote at: $x = -4$.**

There are no horizontal asymptotes since the numerator has a higher degree than the denominator.

There is a **slant asymptote** because the numerator has a degree one higher than the denominator. We can find the equation of the slant asymptote by dividing the numerator by the denominator. Let's use synthetic division for that:

$$\begin{array}{r|rrr} -4 & 3 & -8 & 12 \\ & & -12 & -26 \\ \hline & 3 & -20 & \text{xx} \end{array}$$

So, there is a slant asymptote at: $y = 3x - 20$

19) Graph $g(x)$ and fill out all the information in the table. Write "none" as applicable. $g(x) = \frac{2x^2+2x-4}{x^2+3x-4}$

VA (if any): $\mathbf{x = -4}$	HA (if any): $\mathbf{y = 2}$
x −int (if any) $\mathbf{(-2, 0)}$	y −int (if any): $\mathbf{(0, 1)}$
Hole (if any) $\mathbf{\left(1, \frac{6}{5}\right)}$	Slant asymptote (if any): **none**
Domain: $\{\mathbf{x} \in \mathbb{R} \mid \mathbf{x} \neq \mathbf{-4, 1}\}$	Range: $\{\mathbf{y} \in \mathbb{R} \mid \mathbf{y} \neq \mathbf{\frac{6}{5}, 2}\}$

$g(x)$ is a rational function, so we must begin by factoring it to the extent possible:

$$g(x) = \frac{2x^2+2x-4}{x^2+3x-4} = \frac{2(x^2+x-2)}{(x-1)(x+4)}$$
$$= \frac{2(x-1)(x+2)}{(x-1)(x+4)} = \frac{2(x+2)}{(x+4)}$$

- **There is a hole at:** $\mathbf{x = 1}$ because the term $(x-1)$ is eliminated from the denominator in the simplifying process. The y-value at this point is determined from the simplified form of $g(x)$:

$$y = \frac{2(x+2)}{(x+4)}\bigg|_{x=1} = \frac{2(1+2)}{(1+4)} = \mathbf{\frac{6}{5}}$$ **A hole exists at the point:** $\mathbf{\left(1, \frac{6}{5}\right)}$

- There is a vertical asymptote wherever the factored and simplified denominator is 0. So, **there is a vertical asymptote at:** $\mathbf{x = -4}$**.**

There is a **horizontal asymptote** at the limiting value of $g(x)$ as x approaches $\pm\infty$ because the numerator and denominator of the rational function have the same degree. An easy way to determine the location of the asymptote is to ignore all terms in $g(x)$ except those with the highest exponent:

Horizontal asymptote at: $g(x) = \frac{2x^2 \cancel{+2x-4}}{x^2 \cancel{+3x-4}} = \frac{2x^2}{x^2} = 2$ So, at: $\mathbf{y = 2}$.

There is **no slant asymptote** because the degree of the numerator is not one more than the degree of the denominator.

The **domain** is easy to find for a simple rational function like this one. In this case, the domain is all real numbers except the x-values of any holes and vertical asymptotes. That is: $\{\mathbf{x} \in \mathbb{R} \mid \mathbf{x} \neq \mathbf{-4, 1}\}$.

The **range** is not always easy to find, but for a simple rational function like this one, the range is all real numbers except the y-values of any holes and horizontal asymptotes. That is: $\{\mathbf{y} \in \mathbb{R} \mid \mathbf{y} \neq \mathbf{\frac{6}{5}, 2}\}$.

To determine the **x-intercepts**, set the simplified version of $g(x)$ to zero and solve. The simplified version of $g(x)$ is zero when its numerator is zero, so let's work with just the numerator.

$2(x+2) = 0 \quad \rightarrow \quad x = -2$, so $\mathbf{(-2, 0)}$ **is an x-intercept.**

To determine the **y-intercepts**, set $x = 0$ in the simplified version of $g(x)$ to zero and solve.

$y = \frac{2(x+2)}{(x+4)}\Big|_{x=0} = \frac{2(0+2)}{(0+4)} = 1 \quad \rightarrow \quad y = 1$, so $\mathbf{(0, 1)}$ **is a y-intercept.**

20) Solve and graph the solution on the provided number line: $\dfrac{x-4}{x+5} \leq 0$.

Problems of this nature can be solved by setting up regions on the number line that divide at locations where the individual terms are equal to 0.

For this problem, the divisions would occur at $x = 4$ and $x = -5$, giving rise to regions A, B, C as shown.

In this problem, Region A is $(-\infty, -5)$, Region B is $(-5, 4)$, and Region C is $(4, \infty)$, all of which are open intervals.

Test each region to see if they make the given inequality true or false.

Pick any value in a region for that region's test. If one value in a region satisfies the inequality, then all values in that regions will satisfy the inequality. Any value in a region will suffice for testing purposes.

Region A: Let $x = -6$. $\dfrac{x-4}{x+5} = \dfrac{-6-4}{-6+5} = \dfrac{-10}{-1} = 10 > 0$, so Region A does not satify the given inequality.

Region B: Let $x = 0$. $\dfrac{x-4}{x+5} = \dfrac{0-4}{0+5} = \dfrac{-4}{5} < 0$, so Region B does satify the given inequality.

Region C: Let $x = 5$. $\dfrac{x-4}{x+5} = \dfrac{5-4}{5+5} = \dfrac{1}{10} > 0$, so Region C does not satify the given inequality.

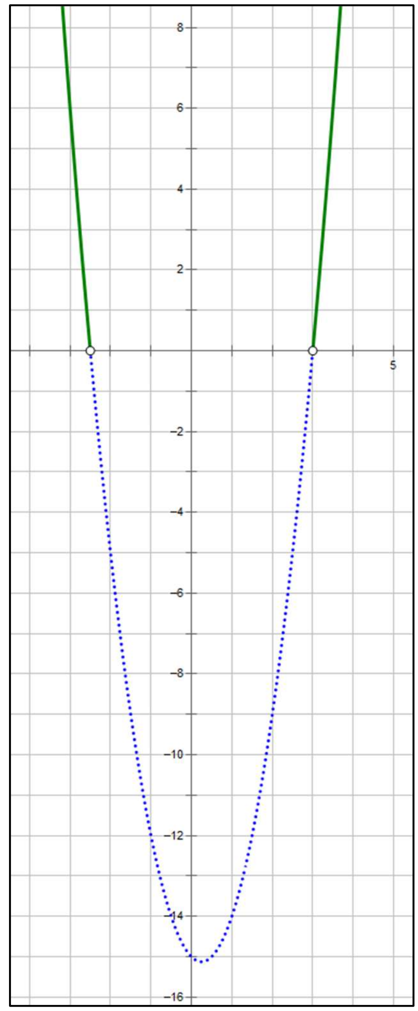
We also need to check to see if the dividing points satisfy the given inequality because of the equal sign that is included in the inequality $(\leq)$. If there were no equal sign in the inequality, we would not need to check the dividing points.

Point $x = -5$. $\dfrac{x-4}{x+5} = \dfrac{-5-4}{-5+5} = \dfrac{-9}{0}$ is undefined, so $x = -5$ does not satify the given inequality.

Point $x = 4$. $\dfrac{x-4}{x+5} = \dfrac{4-4}{4+5} = \dfrac{0}{9} = 0$, so $x = 4$ does satify the given inequality.

Conclude: Region B $(-5, 4)$ and $x = 4$ satisfy the given inequality. Putting these together, we have the solution set: $\mathbf{(-5, 4]}$.

21) Solve the inequality, and graph the solution set on the provided number line: $2x^2 - x > 15$



This problem is much like the last one but easier because we are familiar with the behavior of quadratic functions. Let's start by manipulating the inequality:

$2x^2 - x > 15$

$2x^2 - x - 15 > 0$

$(2x + 5)(x - 3) > 0$

The x-intercepts are: $x = -\frac{5}{2}, 3$.

The question for us becomes whether the quadratic function is positive between the vertices or on the outsides of the vertices, since those are the only possibilities. See the diagram to the right.

Since the quadratic function $y = 2x^2 - x - 15$ opens up, the positive portions of the function must on the outsides of the vertices, i.e., the green portion of the graph.

Therefore, the solution set to this problem is $\left(-\infty, -\frac{5}{2}\right) \cup (x > 3)$.

Endpoints do not need to be checked because there is no equal sign in the inequality being solved.